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Metal-insulator transition in two-dimensional systems with long-range correlated disorder

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Abstract. We study the localization properties of electrons in a two-dimensional model with on-site energies exhibiting long-range correlated disorder. The localization length and conductance of the system are calculated by using the finite size scaling method combined with transfer matrix technique. In the presence of long-range correlations, we find that there is a continuous line of fixed points indicating that the system undergoes a disorder driven Kosterlitz-Thouless-type metal-insulator transition.

PACS. 72.15.Rn Electronic conduction in metals and alloys – 71.30.+h Metal-insulator transitions and other electronic transitions – 73.20.Fz Weak or Anderson localization

1 Introduction

The phenomena of Anderson localization of electrons and the disorder-induced metal-insulator transition (MIT) have been studied extensively for decades [1,2] and continue to attract much attention. Scaling arguments [3] predict that all eigenstates of noninteracting particles are localized in one-dimensional (1D) and two-dimensional (2D) systems for any amount of disorder. However, by introducing physical factors which break the time reversal symmetry, 2D systems can undergo a MIT. For example, in the presence of a uniform magnetic field, numerical calculations indicate that delocalized states appear at the centers of the Landau bands and the critical disorder and the critical exponents for Anderson transition can be determined [4]. On the other hand, in the presence of the spin-orbit couplings, a 2D disordered system is expected to display an Anderson transition [5]. Moreover, metallic phase and MIT has been reported in 2D electron and hole systems even in the presence of time-reversal symmetry [6]. These findings have further intensified the efforts to look for possible mechanism that allow a MIT in systems with spatial dimension $d < 3$.

Another class of systems exhibiting extended states for noninteracting particles in dimension $d < 3$ can be represented by models that in contrast to the traditional Anderson model include correlations in disorder distribution. For instance, by introducing short-range correlated disorder, extended states can be observed at particular resonance energies in one-dimensional (1D) systems [7]. These results were used to explain the MIT observed in doped polyaniline [8] as well as the anomalous transport properties of certain random semiconductors superlattices [9]. Furthermore, it has been demonstrated that long-range correlations in random on-site energies can induce delocalization of 1D electron states [10]. More recently, Carpena, *et al.* [11] studied the localization properties of 1D random binary alloy model in the presence of power-law long-range correlations. It has been found that the 1D system can undergo a correlation-induced MIT once the correlation strength is stronger than a certain threshold. The origin of the MIT in 1D has been analyzed in details in reference [12]. On the other hand, the influence of long-range disorder on the electron motion in 2D systems has been recently investigated [13] and a 2D layered media with long-range correlated disorder in one direction has been shown to exhibit Kosterlitz-Thouless (KT) type MIT [14].

In the present paper, we study how the localization behavior of a 2D system is affected by long-range correlation of disorder. This is of particular interest because 2D is the critical dimensionality in the scaling theory. We generate long-range correlated random on-site energies ϵ_i in 2D systems using the modified Fourier filter method, characterized by a specified spectral density [15] $S(\mathbf{q}) \propto |\mathbf{q}|^{-\alpha}$, where $S(\mathbf{q})$ is the Fourier transform of the two point correlation function $\langle \epsilon_i \epsilon_j \rangle$ and $\alpha = (2-\gamma)/2$ describes the strength of correlation with γ being the correlation exponent. We calculate the localization length and the conductance of such systems for different values of α by the finite-size scaling method combined with the transfer matrix technique. In the presence of long-range correlations ($0 < \alpha < 1$), we find that there exists a line of fixed points, for which the conductance is invariant under the change

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of the sample size, indicating that the system undergoes a disorder-driven KT-type MIT.

The paper is organized as follows: In Section 2 we present the model and the basic formalism, in Section 3 the numerical results are illustrated, and the last section is devoted to brief summary and discussion.

2 Model and formalism

We consider a 2D tight-binding Anderson model on a square lattice

$$H = \sum_i \epsilon_i |i\rangle\langle i| + \sum_{\langle ij \rangle} t|i\rangle\langle j|, \tag{1}$$

where $\langle ij \rangle$ refers to nearest neighbor sites with the hopping integral t taken as the unit of energy ($t = 1$), and $\{\epsilon_i\}$ are random on-site energies, assumed to be long-range correlated and distributed within an interval $[-W/2, W/2]$. We generate the random on-site energies with long-range correlations using the modified Fourier filter method [11,15], which is characterized by a power-law spectral density of the form

$$S(\mathbf{q}) = \frac{2\pi}{\Gamma(1-\alpha)} \left(\frac{|\mathbf{q}|}{2}\right)^{-\alpha} K_\alpha(|\mathbf{q}|), \tag{2}$$

where $\mathbf{q}$ is the wave vector, $\alpha = (2-\gamma)/2$ with γ being the correlation exponent, $K_\alpha(q)$ is the modified Bessel function of order α, and Γ is the gamma function. α describes the strength of the correlation. For $\alpha = 0$ one recovers the 2D Anderson model with uncorrelated disorder. In the case of $0 < \alpha < 1$, there are long-range correlations in the disorder distribution.

On a square lattice, site i can be labeled by its coordinates in the x and y directions, denoted by integers m and n, respectively. The corresponding Schrödinger equation can then be written as

$$\psi_{m,n+1} = (E - \epsilon_{m,n})\psi_{m,n} - \psi_{m-1,n} - \psi_{m+1,n} - \psi_{m,n-1}. \tag{3}$$

where E is the energy of electrons, $\psi_{m,n}$ denotes the amplitude of wavefunction at site (m, n).

In order to calculate the localization length of electrons, we use the finite size scaling method combined with the transfer-matrix technique[2]. We calculate the damping of wave functions in the y direction for a long strip of size $M \times L$ with L being extremely large. The periodic boundary condition is adopted in the x direction. For a given energy E, a $2M \times 2M$ transfer matrix T_n can be easily set up, mapping the wave-function amplitudes at column $n-1$ and n to those at column $n+1$ in the strip. The propagation along the strip is therefore described by the product of transfer matrices

$$Q_L = \prod_{n=1}^{L} T_n. \tag{4}$$

Transfer matrix of equation (4) has M pairs of eigenvalues whose logarithms correspond to the Lyapunov exponents of wave functions, $(\gamma_i, -\gamma_i)\{i = 1, 2, \ldots, M\}$.[2] The largest localization length $\lambda_M(E)$ for energy E in a system with finite width M is given by the inverse of the smallest Lyapunov exponent and the conductance in units of e^2/h can be calculated as [2]

$$G = \sum_{i=1}^{M} \frac{2}{\cosh^2 \gamma_i L}. \tag{5}$$

In our numerical calculation, we choose the length of the strip L to be over 10^6 so that the self-averaging effect automatically takes care of the statistical fluctuations. We estimate and control these fluctuations from the deviations of the calculated eigenvalues of two adjacent iterations which show satisfactory suppression by increasing L. The finally obtained data have statistical errors less than the symbol size in the corresponding figures, so no explicit error bars are indicated.

We use the standard one-parameter finite-size scaling ansatz [2] to obtain the thermodynamic localization length ξ. According to the one-parameter scaling theory, the rescaled localization length λ_M/M can be expressed in terms of a universal function of M/ξ, *i.e.*,

$$\frac{\lambda_M(E)}{M} = f(\frac{M}{\xi(E)}), \tag{6}$$

where $f(x) \propto 1/x$ in the thermodynamic limit $M \to \infty$ for localized states while approaching a constant (~ 1) when ξ diverges.

3 Numerical results and discussions

We first study the localization behavior of the system at the band center $E = 0$ for typical values of α. In Figure 1, we plot the rescaled localization length $\Lambda_M = \lambda_M/M$ as a function of M for $\alpha = 0$, which corresponds to uncorrelated disorder. Λ_M decreases with increasing M, indicating that all states are localized with $W_c = 0$. The picture becomes qualitatively different for on-site energies with long-range correlations. In Figure 2a we show the same curves as in Figure 1 but with the strength of correlation $\alpha = 0.2$. The striking difference from Figure 1 is that the rescaled localization length Λ_M is independent of M in the range of $W < W_c$ with $W_c \simeq 2.8$. This shows that all the points for $W < W_c$ are critical points. It is well known from the finite-size scaling studies that a single critical point is expected with an M-independent fixed point value Λ_c for a conventional continuous transition. Two examples for such a transition from localization to delocalization are the 3D Anderson model [2] and its 2D version with spin-orbital interaction [5]. The MIT shown in Figure 2a is quite different from the conventional MIT in the sense that there is a set of fixed points for $W < W_c$, indicating that the system undergoes a disorder-driven Kosterlitz-Thouless transition [16,17].

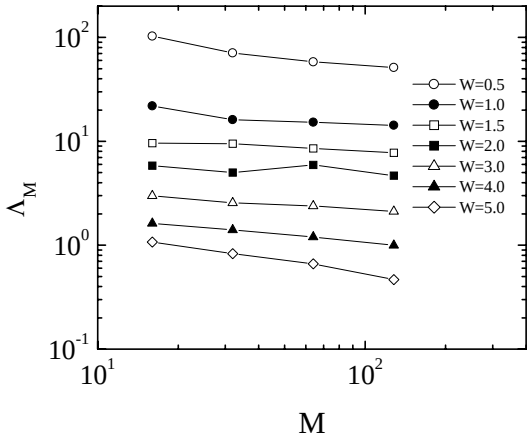

Fig. 1. The rescaled localization length Λ_M as a function of the strip width M at the band center $E = 0$ for $\alpha = 0$.

In Figure 2b we plot the rescaled localization Λ_M as a function of W for different widths M of the system. We observe that all curves merge together for $W < W_c$, which again confirms the presence of the KT transition. In the inset of Figure 2b, we extract the value of localization length ξ in the thermodynamical limit from the scaling ansatz and successfully fit the data with $\xi \propto \exp(\theta/\sqrt{W - W_c})$, indicating the exponentially decay of the localization length on the insulating side. This behavior is typical for the KT transition, providing another evidence of the phase transition of this type. From this fitting we obtain $W_c = 2.74$, and $\theta = 6.5 \pm 0.1$.

It is supposed that the β-function has the property $\beta \propto \mathrm{d}\ln\Lambda_M/\mathrm{d}\ln M$ [2]. Λ_M is independent of M for $W < W_c$ implies that $\beta = 0$ in the metallic or, more precisely, critical regime. Thus, it is expected that the conductance G of a finite square system is also independent of the system size. This is just what we observe in our calculation for the conductance. In Figure 2c we show the calculated conductance G of a square system as a function of W for different sizes M. It can be seen that the curves of G for different M also merge together and are independent of M for $W < W_c$, consistent with our localization length calculations.

We expect that W_c should increase with increasing α because the extended states are more favorable in this situation. In Figure 3, we show the phase diagram in the $\alpha - W_c$ plane at $E = 0$. The states in region I are extended whereas the states in II are localized. One can see that the critical disorder W_c increases with increasing the strength of correlation α, just as we expect.

We show further numerical support for the KT transition by plotting Λ_M *versus* α for fixed disorder. Presented in Figure 4 is the rescaled localization length Λ_M as a function of α for different widths M of the system with $W = 5$. We see that all curves merge together for $\alpha > \alpha_c$ with $\alpha_c \simeq 0.65$. In the metallic regime ($\alpha > \alpha_c$), the conductance G is independent of sample width (inset of Fig. 4).

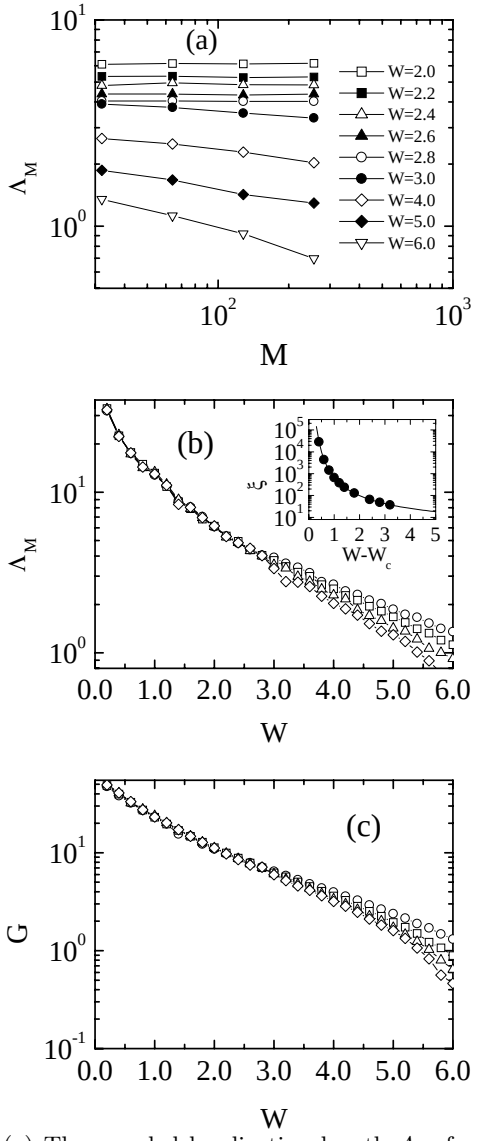

Fig. 2. (a) The rescaled localization length Λ_M for the strip system calculated for $M = 32, 64, 128$, and 256 at the band center $E = 0$ for $\alpha = 0.2$. (b) Λ_M as a function of disorder strength W for ○: $M = 32$, □: $M = 64$, △: $M = 128$, and ◇: $M = 256$. Inset: Fit the thermodynamic localization length ξ with the KT transition. Symbols are numerical data. (c) The conductance G as a function of W.

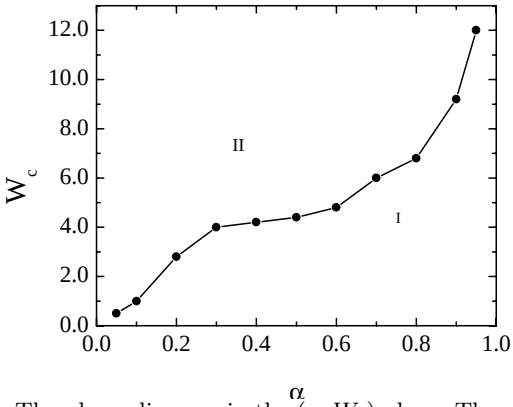

Fig. 3. The phase diagram in the (α, W_c) plane. The states in regimes I and II are extended and localized, respectively.

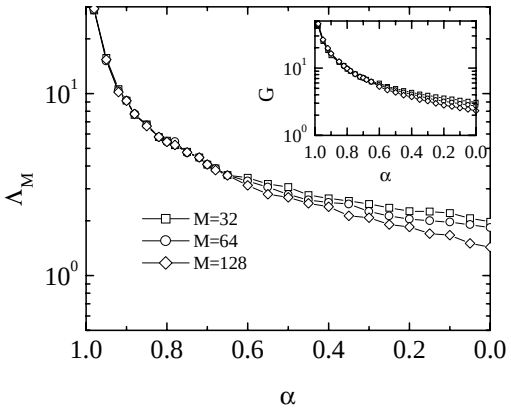

Fig. 4. Λ_M as a function of correlation strength α for $W = 5$. Inset: The conductance G as a function of α.

4 Summary

In summary, we have studied the localization properties of electrons in a 2D system with long-range correlated disorder using the well-developed transfer-matrix method. It is found that the presence of long-range correlation in disorder distribution can induce a KT-type metal-insulator transition. This result is in remarkable contrast to what one would expect for 2D systems with uncorrelated disorder, in which all states are localized for any amount of disorder.

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